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Comparison among Different Models in Determining Optimal Portfolio: Evidence from Dhaka Stock Exchange in Bangladesh

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Abstract: *The main purpose of this paper is to scrutinize the performance of the portfolios under Markowitz, Sharpe's Single-Index Model (SIM), and Constant Correlation Model (CCM) in case of constructing an optimal portfolio and find out which one works best than that of others. For this purpose the monthly closing prices of 238 companies listed in Dhaka Stock Exchange (DSE) and DSE Broad Index ("DSEX") for the period of Jan 2013 to Dec 2014 have been considered. The Markowitz model constructs an optimum portfolio consists of thirteen stocks selected out of 238 stocks, giving the return of 5.20%. On the other hand, Sharpe's single-index model takes thirty two stocks to form an optimum portfolio, giving the return of 4.93%. And the Constant Correlation Model forms an optimum portfolio consists of thirteen stocks, giving the return of 5.49%. Furthermore, the performance of optimal portfolios under various models is evaluated through Sharp ratio, Treynor ratio,* M^2*, and Jensen's Alpha. And the results show that Constant Correlation model outperforms than that of others. The findings of this paper will be useful for policy makers, all kinds of investors, corporations, and other financial market- participants.*

Keywords: *Optimum Portfolio, Markowitz model, Single-Index Model, Constant Correlation Model, Dhaka Stock Exchange (DSE)*

1. Introduction

One of the biggest challenges faced by investors is to decide how to invest for future needs. Risk and return are two distinct factors in investment decision. A rational investor makes investment decisions based on the risk-return trade-off, maximizing return for the same risk, and minimizing risk for the same return. This is done through the construction of portfolio of assets which is subject to the investor's portfolio. Modern portfolio theory is based on the research work of Harry Markowitz about the risk-reduction benefits of diversification in early 1950s. Varian (1993) concisely reviewed the history of modern portfolio theory and stated that implementation of Markowitz model is much more time-consuming and more complex by the number of estimates required. Scherer (2002) addressed that Markowitz's model requires estimations of the entire $k \times k$ variance- covariance matrix. Estimations can be inefficient when the number of asset k is large. Consequently, our ability to measure might be restricted by computational power.

Sharpe's (1963) Single Index Model (SIM) was developed in response to this problem. It assumes correlations with a common index to be the only source of covariance among stock returns. The necessary number of estimations is much reduced, since only the correlations with the index need be considered.

On the other hand, Bernard (1987) provided the evidence that correlations with a common index will not capture the total covariance among stock returns. An alternative simple structure is the

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Constant Correlation Model (CCM), uses the historical average correlation coefficient for the future. Elton and Gruber (1973) and Elton, Gruber, and Urich (1978) showed that, despite its simplicity, the CCM produces better forecasts of the future correlation matrix than those obtained from the SIM or the full historical correlation matrix. The superior performance of the CCM should make it an attractive alternative to the SIM. Although simple portfolio selection procedures have been developed for the CCM by Elton, Gruber, and Padberg (1976, 1977a, b), the computational problem will still be large if the full correlation matrix needs to be estimated to compute the average correlation.

The study's significance arises from the fact that the application of theoretical framework of portfolio management on a real world scenario develops an offer to investors to form a well-balanced optimized and diversified portfolio of stocks in the Dhaka Stock Exchange (DSE). And it can be stated that most of the studies were based on Markowitz model, Single-Index Model or their comparison, only a few numbers of studies were considered constant correlation model in determining optimal portfolio and comparing with other models. This is the main motivation to incorporate constant correlation model to this studies to make an investment decision and find out whether it outperforms or not than that of other popular methods. The study attempts to present an empirical examination of three of the more popular mathematical portfolio selection models: Markowitz model, Single index model, and Constant correlation model. And then the results of these three models are compared in an attempt to ascertain differences in their behavior and find out which one works best in case of determining optimal portfolio. The comparison of these models is very important for investors to choose the appropriate one to construct their portfolios.

1.1 Problem Statement

The problem statement for this research is to assist investors to make their investment decision using the appropriate portfolio model that outperforms than that of others.

1.2 Objectives of the Study

The study has been conducted to compare the results of three financial models: Markowitz model, Single index model, and Constant Correlation model in determining optimal portfolio considering no short-sales and to select the right one. And the study has been conducted on individual securities listed in Dhaka Stock Exchange (DSE). The objectives of this study are:

a. Risk-return analysis of individual securities listed in DSE.

b. Allocate investment in different stocks considering risk-return criteria.

c. Construct optimal portfolio using Markowitz model, Single index model, and Constant correlation model

d. Compare the outcomes of these three models

e. Assist investors in portfolio selection process to make the right choice.

1.3 Structure of the Paper

The text is divided into six parts: Part One, 'Introduction', introduces the importance of trade-off between risk and return. Hence, background of the problem was given briefly in this part; followed by Problem Statement, Objectives of the Study of this research. Part Two, 'Literature review', discusses various studies on portfolio Analysis. Part Three, 'Methodology and Data', explains data source and methodology. Part Four, 'Data Analysis and Findings', discusses the results of the study. Part Five, 'Conclusion', concludes the research result as well as the limitation of the research. Part Six, 'References', provide the lists of full bibliographical details and their journal titles.

2. Literature Review

Markowitz (1952 and 1959), the pioneer of portfolio analysis, affirmed that investors are basically risk-averse. This means that investors must be given higher returns in order to accept higher risk. Markowitz then developed a model of portfolio analysis. The three highlights of this model are normally; the two relevant characteristics of a portfolio are its expected return and some measure of the dispersion of possible returns around the expected return; rational investors will be chosen to hold efficient portfolios, those that maximize expected returns for a given level of risk or, alternatively, minimize risk for a given level of return.

Markowitz (1952) and Tobin (1958) showed that it was possible to identify the composition of an optimal portfolio of risky securities, given forecasts of future returns and an appropriate covariance matrix of share returns. Farrell (1997) stated that it is theoretically possible to identify efficient portfolios by analyzing of information for each security on expected return, variance of return, and the interrelationship between the return for each security and for every other security as measured by the covariance.

Bowen (1984) noted that the Markowitz model required large volumes of data and found that it was difficult to estimate large number of covariance and concluded that semantic and statistical barriers exist that prevent the average businessman from coming to grips with the approach. Michaud (1989) said that Markowitz optimization is not used more in practice, despite its theoretical success due to the conceptually demanding nature of the theory; the fact that most investment companies are not structured to use a mean-variance optimization approach; and Anecdotal evidence that portfolio managers find the composition of optimized portfolios counter-intuitive. Sarker (2013) stated that Markowitz model implementation is much more time-consuming and more complex by the number of estimates required. The study that follows 164 stocks needs 13366 number of covariance. The sheer number of inputs is staggering.

Sharpe (1963) attempted to simplify the process of data input, data tabulation, and reaching a solution. He also developed a simplified variant of the Markowitz model that reduces data and computational requirements. Although Markowitz model was theoretically elegant its serious limitation was the sophisticated and volume of work was well beyond the Markowitz model. William Sharpe (1964) has given model known as Sharpe Single Index Model (SIM) which laid down some steps that are required for construction of optimal portfolios.

Holhenbalken (1975) avowed that there have been significant advances in computing hardware and in the algorithms to compute portfolio volatility. Analysts are now fully capable of obtaining Markowitz solutions for several hundred securities at a time, which makes the relative computational simplicity of index models less valuable. The latest advances in hardware and software now make it possible to construct dedicated stock portfolios based on all the information in the full covariance matrix of security returns.

Elton, Gruber, and Padberg (1976, 1978) proposed techniques for determining optimal portfolios that give a better understanding of the choice of securities to be included in a portfolio. The calculations are easy to carry out and lead to similar results to those given by the Markowitz model with the complete matrix. These techniques are founded upon the Single Index Model. The method is based on an optimal ranking of the assets, established with the help of the simplified correlation representation model. Haugen (1993) stated that Index models can handle large population of stocks. They serve as simplified alternatives to the full-covariance approach to portfolio optimization. Although the SIM model offers a simple formula for portfolio risk, it also makes an assumption about the process generating security returns. The accuracy of the formula of the SIM model for portfolio variance is as good as the accuracy of its assumption.

As seen by Frankfurter et al., (1976) the SIM approach is based on Markowitz model. However, this approach adds the simplifying assumption that returns on various securities is related only through common relationship with some basic underlying factors. According to this study, under conditions of certainty, the Markowitz and SIM approaches will arrive at the same decision set in the experiment. These results demonstrate that under conditions of uncertainty, SIM approach is advantageous over the Markowitz approach.

Affleck-Graves and Money (1976) concluded that the Markowitz approach produces results which are significantly superior to those obtained using an index model. Thus, in practice, the investor wishing to use a risk-return approach to portfolio selection should strive to apply the basic Markowitz formulation. If this is impossible, an index model may be used, but it is stressed that the results obtained may be overly conservative. However, if the total amount to be invested is very large, thus forcing a low upper bound to be imposed on the amount invested in any security, then the index models may be used with much more confidence.

Haugen and Baker (1990) stated that superior tracking ability in the estimation period does not necessarily imply superior predictive power in future periods and Markowitz model has remarkably high predictive power, at least in tracking annual inflation. Edward et. al., (2005) generalized Markowitz's portfolio selection theory and Sharpe's rule for investment decision. An analytical solution is exhibited to show how an institutional or individual investor can combine Markowitz's portfolio selection theory, generalized Sharpe's rule and Value-at-Risk to find candidate assets and optimal level of position sizes for investment (dis-investment). According to Terol et. al., (2006) Markowitz model is a conventional model proposed to solve the portfolio selection problems by assuming that the situation of stock markets in the future can be characterized by the past asset data. However, it is difficult to ensure the accuracy of this traditional assuming because of the large number of extensions to problems of the traditional portfolio selection. As for SIM model, it includes fuzzy betas obtained not only from statistical data but also from expert knowledge.

Omet (1995) argued that the two models are similar. Also, investors might be able to use the more practical approach in generating their efficient frontiers. In other words the SIM model can be used, which is more practical than the Markowitz model in generating ASE efficient frontier. Paudel and Koirala (2006) tested whether or not Markowitz and SIM models of portfolio selection offer better investment alternatives to Nepalese investors by applying these models to a sample of 30 stocks traded in Nepalese stock market from 1997-2006. Results show that the application of the elementary model developed about a half century ago offered better options for making decision in the choice of optimal portfolios in Nepalese stock market.

In addition, Briec & Kerstens (2009) stated that Markowitz model contributes in geometric mean optimization advocated for long term investments. On the other hand, the SIM models are no longer good approximations to multi period. Bekhet & Matar (2012) stated that there is no significance difference between Markowitz model and Single Index Model and the numbers of stocks in the portfolios do not affect the result when comparing the two portfolio models. Gogajeh, Khoshnevis, and Bonab (2015) showed that Sharp models have the ability of forming the optimum portfolio and the model provides the higher return and lower risk than Markowitz model.

Chan, Korceski and Lakonishok (1999) compared the forecasts produced by a full historical model, a constant correlation model, and factor models ranging from one to ten factors. Their conclusion was that constant correlation model had the lowest mean squared errors. They also compared forecasts of correlation coefficients by examining the ability of alternative forecasting techniques combined with one particular forecast of variances to produce the global minimum variance portfolio with the smallest variance, and found the same ordering of techniques. Thus, despite using all the improvements in modeling that have been produced in the last 30 years, they find that the constant correlation still works best.

Ledoit and Wolf (2004) concluded that shrinking the historical pair-wise correlation towards a model which assumes all correlations are the same (the constant correlation model) produces even better results than shrinking towards the Sharpe single index model. Chen (2013) stated that Markowitz model offers higher return than that of Single index model and constant correlation model. Kam (2006) affirmed constant correlation model produces better estimates of future correlation coefficients than do historical correlation coefficients or those produced from the single index approximation.

3. Methodology and data

3.1 Methodology

3.1.1 Markowitz Model

The basic portfolio model was developed by Harry Markowitz (1952, 1959) who derived the expected rate of return for a portfolio of assets and an expected risk measure. To analyze return and risk characteristic of the stocks, the monthly mean return and standard deviation are calculated. And dividend information is incorporated with the monthly closing price. A single asset or portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers higher expected return with the same (or lower) risk or lower risk with the same (or higher) expected return. The expected return on a portfolio is calculated as follows:

$$E(R_p) = \sum_{i=1}^{n} X_i E(R_i)$$

Where X_i is the fraction of investment in stock *i* and $E(R_i)$ is the expected rate of return on stock *i*.

The risk of a portfolio can be written as:

$$\sigma_p = \sqrt{\sum_{i=1}^{n}(X_i^2 \sigma_i^2) + \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i \neq j}}^{n}(X_i X_j \sigma_{ij})}$$

Where, σij is the covariance between securities *i* and *j*.

The efficient set is determined by finding that portfolio with the greatest ratio of excess return (expected return minus risk- free rate) to standard deviation that satisfies the constraint that the sum of the proportions invested in the assets equals 1. Here 7.49% p.a is considered as risk free rate, R_f based on the 91 days treasury bills rate. In equation form we have:

$$\theta = \frac{E(R_p) - R_f}{\sigma_p}$$

Maximize the objective function subject to the constraint

1. $\sum_{i=1}^{n} X_i = 1$
2. $\sum_{i=1}^{N} (X_i E(R_i)) = E(R_p)$
3. $X_i \geq 0, \qquad i = 1, 2, \ldots, n$

This is a constrained maximization problem. To solve this problem a model was developed in Microsoft Excel and Solver Parameters was used for the mean-variance optimization required to identify Markowitz efficient frontier.

3.1.2 Sharpe's Single Index Model

William Sharpe (1963) studied Markowitz's research and worked on simplifying the calculations in order to develop a practical use of the model. There is one key assumption that makes single index model differentiates from other models used to describe the covariance structure. The assumption is that E (eiej) = 0 for all i and j. This implies that the stock prices move together and systematically only because of common co-movement with the market. Besides that, there are no effects beyond the market. Another assumption is the market index is unrelated to unique return. The basic equation underlying the single index model is:

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Where R_i is expected return on security i; α_i is intercept of the straight line or alpha co-efficient (Constant); β_i is slope of straight line or beta co-efficient; R_m is the rate of return on market index and e_i is error term.

Under SIM, ranking of the stocks (from highest to lowest) is done on the basis of their excess return to beta ratio to construct an optimal portfolio. The excess return is the difference between the expected return on the stock and the risk free rate such as the rate on a Treasury bill(here 7.49%p.a is considered as risk free rate based on the 91 days treasury bills rate). For the purpose of analyzing risk characteristic of the stocks, systematic risk or beta is calculated. Beta measures how sensitive a stock's return due to its relationship with the return on the market.

$$\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$$

Where σ_{im} in the covariance of the stock i with the market and σ_m^2 is the variance of the market return. To calculate market return DSE Broad Index data is used.

The ranking represents the desirability of any stock's inclusion in a portfolio. The selection of the stocks depends on a unique cut-off rate such that all stocks with higher ratios of $(R_i - R_f)/\beta_i$ are included and all stocks with lower ratios are excluded. This cut-off point is denoted by C*. The highest C_i value is taken as the cut-off point C*.

$$C_i = \frac{\sigma_m^2 \sum_{i=1}^{n} \frac{(R_i - R_f)\beta_i}{\sigma_{ei}^2}}{1 + \sigma_m^2 \sum_{i=1}^{n} \left(\frac{\beta_i^2}{\sigma_{ei}^2}\right)}$$

Where σ_m^2= the variance in the market index and σ_{ei}^2= the variance of a stock's movement that is not associated with the movement of the market index. This is usually referred to as a stock's unsystematic risk.

After determining the securities to be selected, the investor should find out how much should be invested in each security. The percentage invested in each security is

$$X_i = \frac{Z_i}{\sum_{i=1}^{n} Z_i}$$

Where

$$Z_i = \frac{\beta_i}{\sigma_{ei}^2}\left(\frac{R_i - R_f}{\beta_i} \quad C^*\right)$$

The first expression indicates the weights on each security and they sum up to one. The second expression determines the relative investment in each security. The residual variance on each security σ_{ei}^2 plays an important role in determining the amount to be invested in each security.

After determining the weights on each security, alpha on a portfolio, α_p and beta on a portfolio, β_p are calculated in order to find out the portfolio return and risk. The return on investor's portfolio can be represented as

$$R_p = \alpha_p + \beta_p R_m$$

And the risk of the investor's portfolio σ_p as

$$\sigma_p = \sqrt{\beta_p^2 \sigma_m^2 + \sum_{i=1}^{n} X_i^2 \sigma_{ei}^2}$$

3.1.3 Constant Correlation Model

Constant correlation model assumes that pair-wise correlation coefficients of securities are equal. Average of all pair-wise correlation coefficient is used as the estimate for each pair of stocks denoted as ρ. In order to construct an optimal portfolio, ranking of the stocks (from highest to lowest) is done on the basis of their excess return to standard deviation ratio. This ranking represents the desirability of any stock's inclusion in a portfolio. The selection of the stocks depends on a unique cut-off rate such that all stocks with higher ratios of $(R_i - R_f)/\sigma_i$ are included and all stocks with lower ratios are excluded. This cut-off point is denoted by C*. The highest C_i value is taken as the cut-off point C*.

$$C_i = \frac{\rho}{1 - \rho + i\rho} \sum_{i=1}^{n} \frac{R_i - R_f}{\sigma_i}$$

Where ρ is the correlation coefficient- assumed constant for all securities.

After determining the securities to be selected, the investor should find out how much should be invested in each security. The optimum amount to invest in each security is

$$X_i = \frac{Z_i}{\sum_{i=1}^{n} Z_i}$$

Where

$$Z_i = \frac{1}{(1-\rho)\sigma_i}\left[\frac{R_i - R_f}{\sigma_i} - C^*\right]$$

The expected return and risk on a portfolio are calculated as follows:

$$E(R_p) = \sum_{i=1}^{n} X_i E(R_i)$$

$$\sigma_p = \sqrt{\sum_{i=1}^{n} X_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} X_i X_j \sigma_i \sigma_j \rho_{ij}}$$

3.1.4 Portfolio Performance Evaluation

3.1.4.1 Sharpe Ratio

Performance has two components, risk and return. A commonly used measure of performance is the Sharp ratio, which is defined as the portfolio's risk premium divided by its risk:

$$Sharpe\ ratio = \frac{R_p - R_f}{\sigma_p}$$

The Sharpe ratio suffers from two limitations. First, it uses total risk as a measure of risk when only systematic risk is priced. Second, the ratio itself is not informative. To rank portfolios, the Sharpe ratio of one portfolio must be compared with the Sharpe ratio of another portfolio. Nonetheless, the ease of computation makes the Sharpe ratio a popular tool.

3.1.4.2 Treynor Ratio

The Treynor ratio is a simple extension of the Sharpe ratio and resolves the Sharpe ratio's first limitation by substituting beta risk for total risk. The Treynor ratio is

$$Treynor\ ratio = \frac{R_p - R_f}{\beta_p}$$

In addition, the Treynor ratio does not work for negative beta assets-that is, the denominator must also be positive for obtaining correct estimates and rankings.

3.1.4.3 M-Squared (M^2)

M^2 was created by Franco Modigliani and hid granddaughter, Leah Modigliani-hence the name M-Squared. M^2 is an extension of the Sharpe ratio in that it is based on total risk, not beta risk. The idea behind the measure is to create a portfolio (P') that mimics the risk of a market portfolio. The difference in the return of the mimicking portfolio and the market return in M^2, which can be expressed as a formula:

$$M^2 = (R_p - R_f)\frac{\sigma_m}{\sigma_p} - (R_m - R_f)$$

By using M^2, we are not only able to determine the rank of a portfolio but also which, if any, of our portfolios beat the market on a risk-adjusted basis.

3.1.4.4 Jensen's Alpha

Like the Treynor ratio, Jensen's Alpha is based on systematic risk. A portfolio's systematic risk is measured by estimating the market model, which is done by regressing the portfolio's daily return on the market's daily return. The difference between the actual portfolio return and the calculated risk-adjusted return is a measure of the portfolio's performance relative to the market portfolio and is called Jensen's alpha. Jensen's alpha is also the vertical distance from the security market line (SML) measuring the excess return for the same risk as that of the market and is given by

$$\alpha_p = R_p - \left[R_f + \beta_p\left(R_m - R_f\right)\right]$$

The sign of α_p indicates whether the portfolio has outperformed the market. If α_p is positive, then the portfolio has outperformed the market; if α_p is negative, then the portfolio has underperformed the market. Values of α_p can be used to rank the performance of the portfolios.

At first, the three financial models: Markowitz model, Single index model, and Constant Correlation model are applied to determine an optimal portfolio considering no short-sales in Dhaka Stock Exchange (DSE) and then the performance of optimal portfolios under various models is evaluated through Sharp ratio, Treynor ratio, M^2, and Jensen's Alpha. And finally the outcomes are compared to find out which one works best than that of others.

3.2 Data Source

This paper aims at constructing an optimal portfolio by using Markowitz model, Sharpe's single-index model, and Constant correlation model and thereby comparing the results of these models to find out which model outperforms than that of others in determining optimal portfolio. . For this purpose monthly closing price of the shares, dividend information and monthly closing index value of the benchmark market index (DSE *Broad Index*) have been used for the period from Jan 2013 to Dec 2014. They were collected from Dhaka Stock Exchange. Only two years (Jan 2013 to Dec 2014) monthly data are considered due to the inducement of new indices named DSE Broad Index ("DSEX") and DSE 30 Index ("DS 30") with effect from 28 January, 2013 in DSE. This study takes 238 companies listed in Dhaka Stock Exchange (DSE). The study has used secondary data because it pertains to historical analysis of reported financial data. Auction of 91 days Treasury bill rate 7.49%p.a has been used as proxy for risk-free rate sourced from "Bangladesh Bank". The collected data were consolidated as per study requirements. Various statistical tools have been used to analyze data through Microsoft Excel software.

Table 1: Sector-wise Percentage of Data Coverage

Name of the Industry	Total Number of Companies	No. of Companies	% of Data Coverage
Bank	30	30	100.00%
Financial Institutions	23	21	91.30%
Engineering	29	23	79.31%
Food & Allied	18	16	88.89%
Fuel & Power	17	14	82.35%
Jute	3	2	66.67%
Textile	40	28	70.00%
Pharmaceuticals & Chemicals	27	20	74.07%
Paper & Printing	2	1	50.00%
Services & Real Estate	4	3	75.00%
Cement	7	6	85.71%
IT - Sector	6	6	100.00%
Tannery Industries	5	5	100.00%
Ceramic Industry	5	5	100.00%
Insurance	46	45	97.83%
Telecommunication	2	2	100.00%
Travel & Leisure	4	2	50.00%
Miscellaneous	11	9	81.82%
Total	**279**	**238**	**85.30%**

From the table 1 it can be seen that among 279 companies 238 companies are selected due to the reason of the availability of data within the time frame (Jan 2013 to Dec 2014). The companies that are listed and traded but stopped operations during the time frame are excluded. It has covered 85.30% data and it can be said that data coverage is moreover satisfactory to make an investment decision.

4. Data Analysis and Findings

4.1 Results of Markowitz Model

A model was developed in Microsoft Excel and Solver Parameters was used for the mean-variance optimization required to identify Markowitz efficient frontier. Under solver parameters about 120 trials are performed to find out the weights of securities considering the above said constraints.

Table 2: Results of Optimum Portfolio Construction under Markowitz Model

No.	Name of Securities	Weight of Securities (%)	$E(R_i)$	σ_i
1	BATBC	27.30%	5.49%	9.52%
2	Berger Paints	11.55%	5.24%	10.75%
3	Lafarge Surma Cement	8.59%	6.42%	13.87%
4	Stylecraft	8.34%	2.84%	18.11%
5	Square Pharma	7.74%	4.30%	12.19%
6	Bangas	7.11%	5.73%	21.35%
7	Eastern Cables	6.92%	4.41%	12.06%
8	Summit Alliance Port Limited	6.09%	7.42%	22.01%
9	Quasem Drycells	5.30%	4.00%	12.72%
10	ACI Limited	3.27%	7.79%	19.34%
11	Bd. Thai Aluminium	3.23%	2.60%	16.39%
12	Pharma Aids	2.33%	2.81%	11.70%
13	Alltex Ind. Ltd.	2.22%	8.87%	37.97%
Theta			**0.9330**	
Portfolio Return			**5.20%**	
Portfolio Risk			**4.91%**	

Table 2 clearly explains the results of empirical analysis. If short sale is not allowed, it is seen that the optimum portfolio consists of only 13 securities with the largest investment in BATBC and the smallest in Alltex Ind. Ltd. Portfolio return and portfolio risk have found out respectively 5.20% and 4.91%. The efficient set is determined by finding that portfolio with the greatest ratio of excess return to standard deviation (here theta is 0.933) that satisfies the constraint that the sum of the proportions invested in the assets equals 1. Such portfolio is the optimum portfolio and the securities included in the portfolio are the efficient securities. In a diversified portfolio, some securities may not perform as expected but others may exceed the expectation to the anticipated one. Higher number of negative covariance indicates more diversification power. Here the number of covariance will be n (n-1)/2 = 13(13-1)/2 = 78. In case of no-short sale scenario, the study has shown 35 numbers of negative covariance that is moreover satisfactory to realize the diversification effect as well as to maximize excess return to standard deviation ratio.

4.2 Results of Single-Index Model

Firstly the securities are ranked according to their excess return to beta ratio from highest to lowest. And then C_i is calculated in order to find out the optimum C_i. The highest C_i value is considered as the optimum C_i. And this is known as the cut-off point C*.

Table 3: Results of determining Cut-off Rate under Single-Index Model (SIM)

Security Name	$(R_i - R_f)/\beta_i$	$[(R_i - R_f)^* \beta_i] / \alpha^2_{ei}$	$\sum[(R_i - R_f)^* \beta_i] / \alpha^2_{ei}$	β_i^2/α^2_{ei}	$\sum\beta_i^2/\alpha^2_{ei}$	C_i
Al-Haj Textile	1.4009	0.0468	0.0468	0.0334	0.0334	0.00017
Anlima Yarn	0.4324	0.0012	0.0479	0.0027	0.0361	0.00017
Marico Bangladesh	0.3250	0.4473	0.4953	1.3763	1.4124	0.00178
AMCL (Pran)	0.1239	0.2299	0.7252	1.8565	3.2689	0.00259
Glaxo SmithKline	0.1195	1.2816	2.0068	10.7252	13.9941	0.00689
BATBC	0.1167	2.4047	4.4115	20.6106	34.6047	0.01415
Stylecraft	0.1140	0.1317	4.5432	1.1551	35.7599	0.01452
Gemini Sea Food	0.1117	0.0435	4.5867	0.3892	36.1491	0.01464
Bangas	0.1084	0.5366	5.1233	4.9491	41.0982	0.01610
Eastern Cables	0.1031	0.9886	6.1119	9.5908	50.6890	0.01865
CVO Petrochemical Refinery Limited	0.0994	0.7601	6.8720	7.6495	58.3385	0.02049
Pharma Aids	0.0937	0.3766	7.2487	4.0212	62.3597	0.02135
Libra Infusions	0.0865	0.3685	7.6172	4.2610	66.6207	0.02216
Alltex Ind. Ltd.	0.0821	0.5890	8.2062	7.1717	73.7923	0.02339
Berger Paints	0.0805	2.5560	10.7622	31.7666	105.5590	0.02813
Samorita Hospital	0.0739	0.4907	11.2529	6.6371	112.1961	0.02891
Bata Shoe	0.0647	1.9010	13.1540	29.3637	141.5598	0.03142
National Tubes	0.0634	1.6688	14.8228	26.3377	167.8975	0.03331
Standard Ceramic	0.0566	0.4396	15.2624	7.7706	175.6681	0.03371
Eastern Lubricants	0.0542	0.5322	15.7946	9.8234	185.4915	0.03414
Square Pharma	0.0517	2.0022	17.7968	38.6911	224.1825	0.03550
ACI Limited	0.0508	3.3478	21.1446	65.9175	290.1000	0.03728
The Ibn Sina	0.0499	0.5983	21.7428	11.9933	302.0933	0.03754
Renata Ltd.	0.0481	1.2379	22.9807	25.7277	327.8210	0.03799
Rangpur Foundry	0.0477	0.7975	23.7782	16.7187	344.5397	0.03825
Lafarge Surma Cement	0.0459	5.4138	29.1920	117.9274	462.4671	0.03947
Olympic Industries	0.0451	2.6145	31.8065	57.9215	520.3886	0.03988
Usmania Glass	0.0449	0.8369	32.6435	18.6366	539.0252	0.04000
National Polymer	0.0440	1.0351	33.6786	23.5444	562.5696	0.04011
Modern Dyeing	0.0424	0.5538	34.2324	13.0697	575.6394	0.04014
Quasem Drycells	0.0413	2.0052	36.2376	48.6014	624.2407	0.04020
Ambee Pharma	0.0410	0.4120	36.6496	10.0517	634.2924	0.04021

From the table 3 it can be seen that among 238 companies the optimum portfolio consists of investing in 32 companies for which $(R_i - R_f) / \beta_i$ is greater than a particular cut off point C*. Here, the cut-off rate is 0.04021.

Table 4: Results of Optimum Portfolio Construction under SIM

No.	Name of Securities	Weight of Securities (%)	Mean Return (%)	Beta
1	BATBC	15.21%	5.49%	0.4172
2	Marico Bangladesh Ltd.	11.62%	5.04%	0.1357
3	Berger Paints	8.97%	5.24%	0.5739
4	Glaxo SmithKline	8.58%	5.39%	0.3988
5	Eastern Cables	6.61%	4.41%	0.3672
6	Bata Shoe	5.21%	4.23%	0.5561
7	Al-Haj Textile	4.14%	6.81%	0.0441
8	Pharma Aids	3.71%	2.81%	0.2330
9	National Tubes	3.09%	5.67%	0.7956
10	AMCL (Pran)	3.07%	3.15%	0.2036
11	Bangas	2.89%	5.73%	0.4707
12	Samorita Hospital	2.54%	3.25%	0.3544
13	Square Pharma	2.53%	4.30%	0.7104
14	CVO Petrochemical Refinery Ltd.	2.24%	8.69%	0.8120
15	Libra Infusions Limited	2.15%	3.82%	0.3697
16	Lafarge Surma Cement	2.14%	6.42%	1.2616
17	ACI Limited	1.99%	7.79%	1.4113
18	Stylecraft	1.77%	2.84%	0.1942
19	Standard Ceramic	1.48%	2.58%	0.3461
20	Eastern Lubricants	1.26%	3.00%	0.4386
21	The Ibn Sina	1.25%	2.48%	0.3725
22	Rangpur Foundry	1.24%	2.57%	0.4073
23	Alltex Ind. Ltd.	1.21%	8.87%	1.0040
24	Renata Ltd.	1.01%	4.54%	0.8127
25	Gemini Sea Food	0.97%	1.92%	0.1159
26	Olympic Industries	0.74%	7.67%	1.5601
27	National Polymer	0.69%	2.90%	0.5181
28	Anlima Yarn	0.58%	0.95%	0.0074
29	Usmania Glass	0.55%	3.49%	0.6378
30	Quasem Drycells	0.25%	4.00%	0.8182
31	Modern Dyeing	0.20%	2.99%	0.5570
32	Ambee Pharma	0.11%	1.79%	0.2830
Beta on a Portfolio, β_p			**0.4486**	
Alpha on a Portfolio, α_p			**0.0454**	
Portfolio Return			**4.93%**	
Portfolio Risk			**4.16%**	

Table 3 and 4 clearly explain the results of empirical analysis. Only those securities are desirable in the portfolio, which have positive excess return over risk free return. If short sale is not allowed, it is seen that the optimum portfolio consists of only 32 securities with the largest investment in BATBC And the smallest in Ambee Pharma. Portfolio return and portfolio risk have found out respectively 4.93% and 4.16%. The result shows that portfolio beta is significantly lower than the market beta and portfolio return is much higher than the portfolio variance.

4.3 Results of Constant Correlation Model

Firstly the securities are ranked according to their excess return to standard deviation ratio from highest to lowest. And then average correlation coefficient, ρ is calculated considering all pair-wise correlation coefficients. And C_i is calculated in order to find out the optimum C_i. The highest C_i value is considered as the optimum C_i. And this is known as the cut-off point C*.

Table 5: Results of Optimum Portfolio Construction under CCM

Security Name	R_i	σ_i	$(R_i - R_f)/\sigma_i$	$\sum(R_i - R_f)/\sigma_i$	$\rho/(1-\rho+i\rho)$	C_i	Weight (X_i)
BATBC	0.0549	0.0952	0.5110	0.5110	0.2467	0.1261	29.50%
Berger Paints	0.0524	0.1075	0.4295	0.9406	0.1979	0.1861	16.28%
Lafarge Surma Cement	0.0642	0.1387	0.4175	1.3581	0.1652	0.2244	11.49%
Glaxo SmithKline	0.0539	0.1241	0.3840	1.7421	0.1418	0.2470	9.32%
Marico Bangladesh	0.0504	0.1160	0.3804	2.1224	0.1242	0.2636	9.57%
ACI Limited	0.0779	0.1934	0.3706	2.4930	0.1105	0.2754	5.08%
Grameenphone Ltd.	0.0445	0.1055	0.3626	2.8556	0.0995	0.2841	8.33%
Bata Shoe	0.0423	0.1079	0.3336	3.1892	0.0905	0.2885	4.64%
Eastern Cables	0.0441	0.1206	0.3139	3.5030	0.0830	0.2906	2.03%
Olympic Industries	0.0767	0.2254	0.3124	3.8155	0.0766	0.2923	1.00%
National Tubes	0.0567	0.1622	0.3107	4.1262	0.0712	0.2936	1.26%
Summit Alliance Port	0.0742	0.2201	0.3089	4.4351	0.0664	0.2946	0.82%
Square Pharma	0.0430	0.1219	0.3015	4.7366	0.0623	0.2951	0.69%
Beta on a Portfolio, β_p			**0.6619**	**Alpha on a Portfolio, α_p**			**0.0491**
Portfolio Return			**5.49%**	**Portfolio Risk**			**7.69%**

From the table 5 it can be seen that among 238 companies the optimum portfolio consists of investing in 13 companies for which $(R_i - R_f)/\sigma_i$ is greater than a particular cut off point C*. Here, the cut-off rate is 0.2951. If short sale is not allowed, it is seen that the optimum portfolio consists of only 13 securities with the largest investment in BATBC and the smallest in Square Pharma. Portfolio return and portfolio risk have found out respectively 5.49% and 7.69%. The result shows that portfolio beta is significantly lower than the market beta and portfolio return is much higher than the portfolio variance.

4.4 Portfolios Performance Evaluation under various models

Table 6: Measures of Portfolio Performance Evaluation

Ratios	Markowitz	SIM	CCM	Market
Sharpe ratio	0.932994	1.033963	0.631732	0.041964
Treynor ratio	0.071292	0.095945	0.073436	0.002521
M-Squared	0.053528	0.059594	0.035430	0
Jensen's Alpha	0.044173	0.041909	0.046937	0

Table 7: Ranking of Portfolios by Performance Measure

Rank	Sharpe ratio	Treynor ratio	M-Squared	Jensen's Alpha
1	SIM	SIM	SIM	CCM
2	Markowitz	CCM	Markowitz	Markowitz
3	CCM	Markowitz	CCM	SIM
4	Market	Market	Market	Market

From Table 6 and 7 it can be said that all three portfolios have Sharpe and Treynor ratios greater than those of the market, and all three portfolios' M^2 and α_i are positive; therefore investment should be satisfied with their performance. Among three portfolios, Portfolio under Single index model performs much better than that of others when total risk is considered (Higher Sharpe ratio). Similarly, M-Squared is also higher for portfolio under SIM (0.059594) than for others. On the other hand, when systematic risk is used, Treynor ratio is higher for portfolio under SIM (0.095945) than for others, and Jensen's Alpha is higher for portfolio under CCM (0.046937) than for others. But Treynor ratio does not work for negative beta assets to obtain correct estimates and rankings. And out of 238 companies, 13 companies offers negative beta. That's why it can be said that Jensen's Alpha is the best ratio to obtain correct estimates and rankings. And portfolio under Constant Correlation Model (CCM) has done a better job of generating excess return relative to systematic risk than others because portfolio under CCM has diversified away more of the nonsystematic risk than others. And it can also be said that portfolio under CCM outperforms portfolio under SIM. The results are almost similar to the earlier results (e.g. Chen 2013, Ledoit and Wolf 2004).

5. Conclusion

Risk and return play an important role in making any investment decisions. This study aims at analyzing the opportunity that are available for investors as per as returns are concerned and the investment of risk thereof while investing in equity of firms listed in the Dhaka stock exchange. Three popular models: Markowitz model, Sharpe's single-index model, and Constant Correlation model were applied by using the monthly closing prices of 238 companies listed in DSE and DSE broad index for the period from Jan 2013 to Dec 2014. From the empirical analysis, it can be said that data coverage of this paper is moreover satisfactory to make an investment decision because it covers 85.30% of data. Out of 238 companies taken for the study, 21 companies are showing negative returns and the other 217 companies are showing positive returns. Out of 238 companies, 90 companies where market beta is above 1, show that the investments in these stocks are outperforming than the market. And 121 companies offers less return than risk free rate.The study shows that optimum portfolio under Constant Correlation Model (CCM) performs much better than portfolios under Markowitz model and Single-index model (SIM). The results are almost similar to the earlier results (e.g. Chen 2013, Ledoit and Wolf 2004, Kam 2006). From this empirical analysis, to some extent one can able to forecast individual security's return through the market movement and can make use of it.

5.1 Limitations of the Study

This paper attempts to construct an optimal portfolio by using appropriate model which one outperforms others and thereby helps to make investment decisions. The current study however has some limitations. This study did not take into consideration the companies that are not listed on the DSE and the companies that are listed and traded but stopped operations. Only two years (Jan 2013 to Dec 2014) data are considered due to the inducement of new indices with effect from 28 January, 2013 in DSE. That's why this study used monthly data rather than daily data. This study has successfully run three models: Markowitz model, Single-index model, and Constant correlation model in determining an optimal portfolio; future research may concentrate on multi-index model and the development of new portfolio selection models and policies.

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